Up to $n$: pragmatic inference about an unspecified starting point
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The modified numerals up to $n$ and at most $n$ form an interesting pair. Intuitively they seem to have the same semantic content ($\leq n$). However, as Blok (2015) points out, up to $n$ but not at most $n$ triggers a proximity inference and they interact with evaluative adverbs differently. For example, (1a) implies that the number of guests will be close to 100 and the speaker is happy that lots of people will come, whereas (1b) implies that the speaker is happy that not many people will come and does not suggest that the number will be close to 100.

(1) a. Fortunately, up to 100 people will attend my wedding.
    b. Fortunately, at most 100 people will attend my wedding.

On the other hand, the above contrasts disappear when the modified numerals are under the scope of existential deontic modals and the speaker is assumed to be authoritative: both (2a) and (2b) mean that the listener is allowed to borrow from 0 to 5 books.

(2) a. You are allowed to borrow up to five books.
    b. You are allowed to borrow at most five books.

To account for this asymmetric pattern, I propose that up to $n$ generates possibilities (in Coppock & Brochhagen’s (2013) inquisitive semantics framework) from an unspecified starting point $\theta$ all the way “up to” $n$ (3a), and that the listener infers $\theta$ pragmatically by reasoning about the strength of the utterance and the speaker’s uncertainty.

(3) a. $[\text{up to } n] = \{\lambda M(d,t). \max \{d \mid M(d) = f(\theta, n)\} \mid f \text{ is a choice function}\}$, where $0 \leq \theta < n$ is a contextually determined starting point.
    b. $[\text{up to 100 people will attend my wedding}] = \{p_\theta, p_{\theta+1}, \ldots, p_{100}\}$, where $0 \leq \theta < n$ and $p_i$ is the proposition that exactly $i$ people will attend the party.
    c. $[\text{You are allowed to borrow up to five books}] = \{\diamond \{p_\theta, p_{\theta+1}, \ldots, p_5\}\}$, where $0 \leq \theta < n$ and $p_i$ is the proposition that you borrow exactly $i$ books.

When up to $n$ is unembedded (3b), a higher $\theta$ results in fewer possibilities, which makes the utterance more informative. Therefore $\theta$ should be reasonably (given speaker uncertainty) close to $n$. The listener can then infer that the actual number is close to $n$ and that evaluative adverbs target its proximity. When up to $n$ is under an existential deontic modal (3c), however, raising more possibilities (having a lower $\theta$) leads to a stronger meaning due to the free choice inference (e.g., Kratzer & Shimoyama 2002). Hence, if the speaker is assumed to be authoritative, the listener will expect him to assert the strongest utterance to minimize the uncertainty, which means that the preferred $\theta$ is 0 and the full range from 0 to $n$ is allowed.

The idea of integrating unspecified lexical semantics with general pragmatic inference has been applied to various phenomena such as vagueness (e.g., Lassiter & Goodman, 2013) and embedded implicature (Potts, Lassiter, Levy, and Frank, 2015). The proposed analysis illustrates that it can also help explain the intricate inference patterns of modified numerals.

Selected References